Circuit Theorems – Scope and Limitations
(Part III: Miller's Theorem)

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***Abstract* —This is the third and the last one of three parts of the above titled paper. This part will deal with Miller's theorem, how its various versions can be used in analyzing passive and active circuits. Next matrix method of analysis is outlined and it is shown that matrix method is simpler and faster than the Miller's equivalent circuits. Finally, a generalized Miller's theorem is stated, proved and its applications are given.**

Keywords: Miller's theorem, Miller's equivalent circuits, Generalized Miller's theorem, Matrix method of analysis

I. INTRODUCTION

TWO networks are said to be equivalent at a pair of terminals if the voltage-current relationships for the two networks are identical at these terminals. Replacing a part of a complex network by its equivalent circuit helps simplifying the analysis. For example, the star-delta transformation can help analyzing certain circuits where the series-parallel reduction technique alone does not yield the solution [1].

Miller's theorem [2] and its dual [3] are known for a long time. Macnee [4] suggests an alternative presentation to improve the understanding of the same and puts in a word of caution on the prediction of the reverse transmission and output immittance from the equivalent. Later two more versions [5] and [6] of Miller's theorem appeared. Rathore [7] presented a generalized Miller theorem and its applications in the analysis and synthesis of networks. Ki *et al*. [8] examine pole splitting of a generic single-stage amplifier in detail. They emphasize the correct condition for applying Miller's Theorem, identify the actual movement of the poles, derive the input and output impedances, and conclude that the application of the Miller effect should be used with precaution; otherwise, wrong results could be obtained.

Filipkowski [9] suggests a new approach to the problem of loss of information about the poles and zeros in the transfer function introduced by the Miller effect approach. Mazhari [10] clarifies that not only can a reasonable estimate for both dominant and non-dominant poles be obtained through Miller's theorem but additional insight is also offered into pole splitting, not afforded by conventional analysis. Moura [11] provides a detailed and rigorous analysis of Miller's theorem and its dual. He utilizes the iterative process to estimate the closed-loop gain from the open-loop gain due to which the value converges to the true closed-loop gain as the number of iterations (n) tends to infinity. He defines the errors in voltage gain and input admittance after n iterations, highlights them as associated with Miller's approximations. He concludes that the theorems can be applied to solve with high accuracy, certain types of complex circuits and simultaneously provide insights regarding the effects introduced by a feedback loop.

Nayaka [12] utilizes Miller's theorem for analysis of high-frequency voltage amplifier where the approximate value of gain is taken as $A/\sqrt{2}$ instead the mid-band gain value A, to obtain more appropriate results. Palumbo *et al*. [13] extended the use of Miller's theorem and derived generalized Miller formulae for weakly nonlinear networks and applied it to analyze the harmonic distortion of bipolar transistor in CE configuration.

Miller's theorems provide a simple, and yet a powerful tool in simplifying the circuit analysis by decoupling the input and output circuits. The approaches adopted can be classified into two groups based on the solution obtained: the approximate solution in which an approximate value of the gain is assumed [4] [11][12] and, the other where gain is not assumed but calculated exactly [7][14]. In the first group, exact solution can be approached by the process of iteration [11]. Rathore [7] and later Dutta Roy [14] have shown that Miller's theorem and its dual can be used for the exact analysis. In [14], it was demonstrated that, even though there are undetermined gain parameters in the equivalent Miller impedances, they do not act as deterrents and the exact analysis can be carried out. However, they were applied only once or successively and then with the help of other theorems like Thevenin, the networks were simplified to arrive at the final results. Thus Miller's theorem and its dual were not fully exploited to obtain the final results.

Rathore and Shah [15] and Prasad [16] showed that all the four Miller's equivalents can be used not only to one particular element but several elements in succession or simultaneously to both the passive and active circuits. It is observed that if the equivalents are applied in succession, the circuits to be solved are more complex than when applied simultaneously to different elements. The latter approach requires more number of equations to be solved simultaneously than the former one. In all the applications of Miller theorems, the most difficult task is to calculate exactly a particular transfer function as an intermediate step and requires involved algebra.

There are many ways to prove the equivalence of Miller's networks. Proofs have already been derived in terms of network parameters in [7], however, Rathore and Shah have used the substitution theorem [17] to prove them [15].

Matrix method of analysis [18] is given in Section III. It gives the exact solution of the circuit without making any approximation for any transfer function. The classical loop and node methods for passive circuits are the special cases. It gives an insight as to how a reciprocal network can be converted into a non-reciprocal one using controlled sources. No intermediate step for calculating a specific transfer function is required and no special precaution is to be taken for determining any function including output admittance and reverse gain.

Thus we have seen that there is a class of circuits which have controlled sources dependent upon current through or voltage across some passive element in the circuit. However, there is another class of circuits where elements are dependent upon some transfer function (voltage, current, resistance and conductance). These are evolved when a series (parallel) element of a ladder is replaced by two elements dependent upon one of the transfer functions.

Two 2-port network N_1 and N_2 can be connected in four possible feedback connections, namely, parallel-parallel (PP), series-series (SS), parallel-series (PS) and series-parallel (SP) connections [1]. For each connection, there is one particular Miller's theorem known which when applied to N_2 (say), reduces it to an equivalent 2-port consisting of two 2-terminal emittances, one appearing at each port of N_1. However, each of these known theorems deals with a special N_2 network. Rathore deals with four generalized Miller's equivalent circuits[7]; each one is applicable to a particular connection of the general two 2-port network N_2. From these equivalent circuits, a generalized Miller's theorem is stated. Many known results, such as capacitance multiplication, high input impedance of the emitter follower and the Darlington pair, and synthesis of driving point and transfer functions by some network configurations, can be understood/explained through the Miller's theorem.

II. MILLER'S EQUIVALENT CIRCUITS

A. Miller voltage transfer function equivalent circuit

Consider the circuit shown in Fig. 36(a). Here

$$i_A = -i_B \tag{78}$$

Let

$$\frac{v_B}{v_A} = A_v. \tag{79}$$

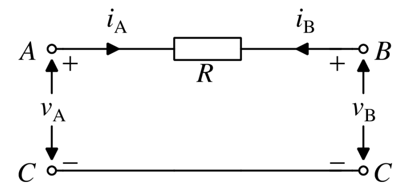
The circuit can be represented as shown in Fig. 36(b) using the substitution theorem. The voltage sources v_A and v_B are replaced by equivalent resistances

$$R_1 = \left(\frac{A_v}{1-A_v}\right)R \quad \text{and} \quad R_2 = \left(\frac{1}{A_v-1}\right)R. \tag{80}$$

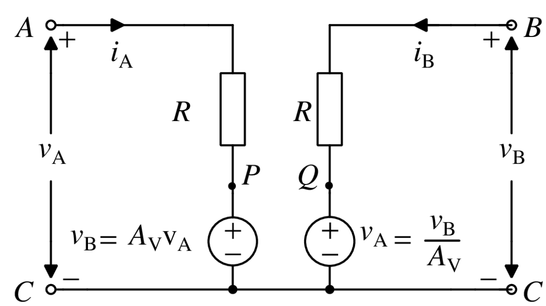
keeping the same potentials at points P and Q as shown in Fig. 36(b). Finally, the two series resistances (R and R_1) and (R and R_2) in Fig. 36(c) are replaced, respectively, by

$$R_A = \left(\frac{1}{1-A_v}\right)R, \qquad R_B = \left(\frac{A_v}{A_v-1}\right)R. \tag{81}$$

Thus, R in Fig. 36(a) can be replaced by two resistances R_A and R_B as shown in Fig. 36(d).



(a)



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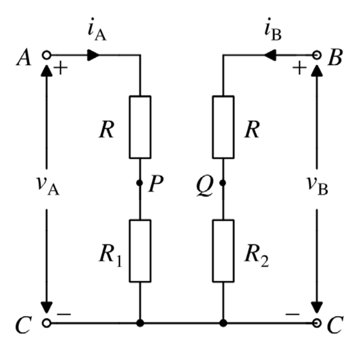
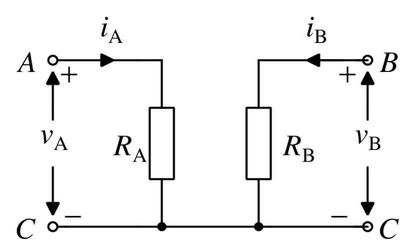
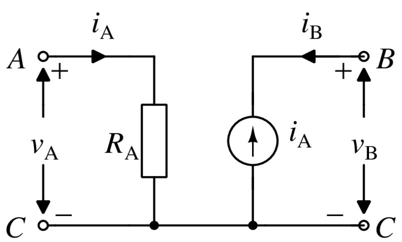


(c)



(d)

Figure numbers and equation numbers are in continuation with the Part 2 of the paper published in volume 9 no. 1, pp.1-12



(*e*)

Figure 36. (*a*) Circuit; (*b*) - (*e*) Equivalent circuits.

By using the substitution theorem, the resistance R_B can be replaced by a current source $i_A = v_A/R_A$ as shown in Fig. 36 (*e*). This Miller's voltage equivalent will be shown to be more convenient to use than Fig. 36(*d*) when applied simultaneously. In general, there are ${}^{N-1}C_2$ possible ways of choosing a resistor in a circuit to which the Miller's voltage equivalent circuit can be applied, where *N* is the number of nodes.

B. Miller transfer resistance equivalent circuit
In Fig. 37(*a*), let

$$\frac{v_B}{i_A} = R_E \tag{81}$$

then the Miller resistance equivalents are shown in Fig. 37(*a*) and (*b*). The values of the Miller's current equivalents are

$$R_A = R + R_E, R_B = -R_E \tag{82}$$

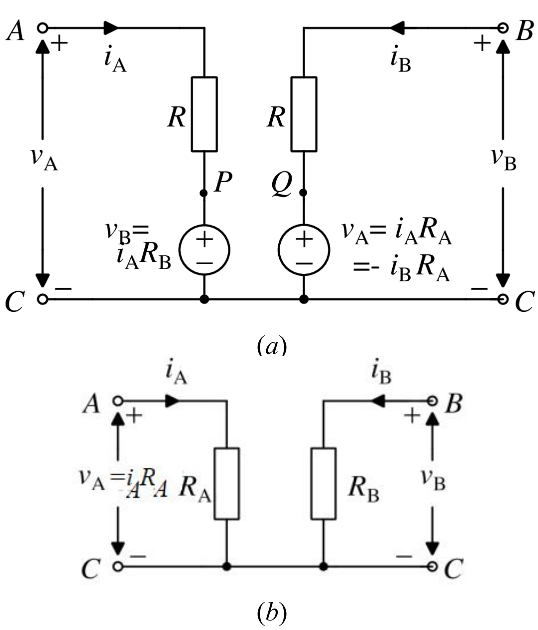


Figure 37. (*a*)-(*b*) Equivalent circuits.

C. Miller current gain equivalent circuit
Consider the circuit shown in Fig. 38(a). Here

$$v_A = v_B. \tag{83}$$

$$\frac{i_B}{i_A} = A_i \tag{84}$$

Fig. 38(*b*) - (*d*) give the Miller's current equivalents. The voltage sources in Fig. 38(*c*) are replaced by the resistances $R_1 = RA_i$ and $R_2 = R/A_i$ keeping the same potentials at points *P* and *Q*. Finally, in Fig. 37(*d*), the two series resistances are replaced by

$$R_A = (1 + A_i)R, \qquad R_B = \left(1 + \frac{1}{A_i}\right)R, \tag{85}$$

Thus, *R* in Fig. 38(a) can be replaced by two resistances R_A and R_B as shown in Fig. 38(*d*). While applying Miller's current equivalents there is an increase in the number of nodes by one. Note that in this equivalent $i_B = 0$ will be a trivial case.

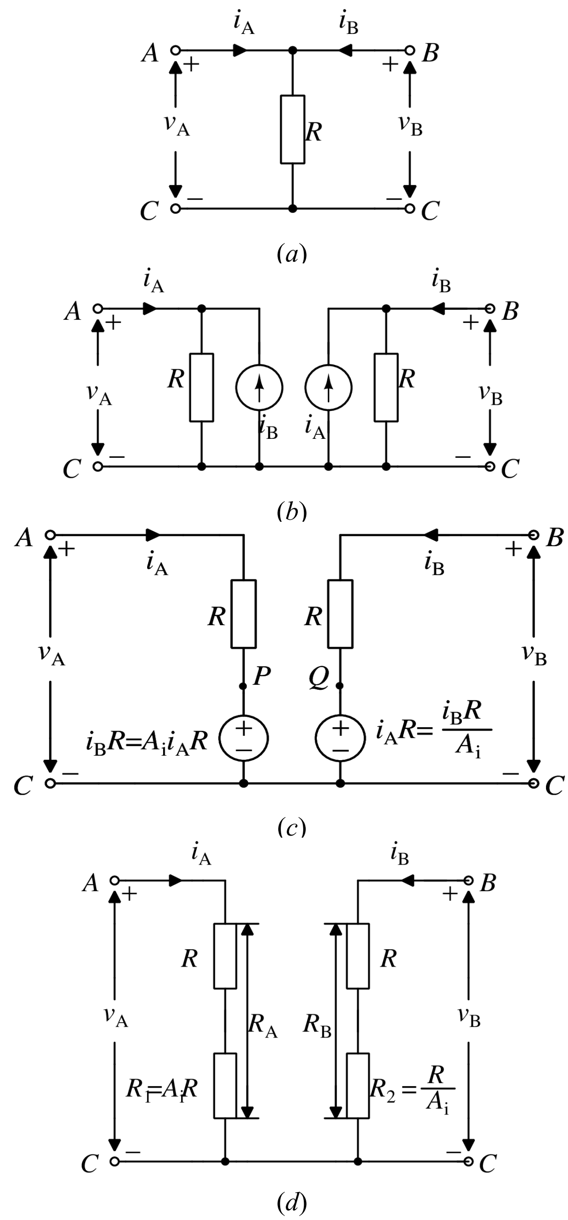
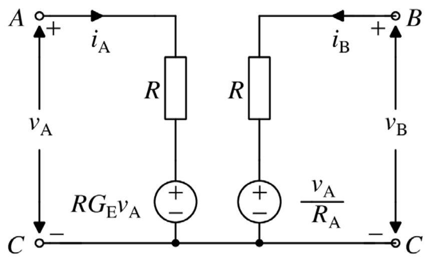


Figure 38. (*a*) Given circuit. (*b*) - (*d*) Equivalent circuits.

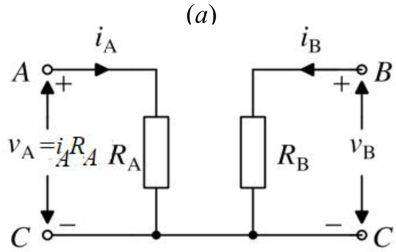
D. Miller transfer conductance equivalent circuit
In Fig. 39(*a*), let

$$\frac{i_B}{v_A} = G_E \tag{86}$$

then the Miller conductance equivalents are shown in Fig. 39(*a*) and (*b*).



(*a*)



(*b*)

Figure 39. (*a*)-(*b*) Equivalent circuits.

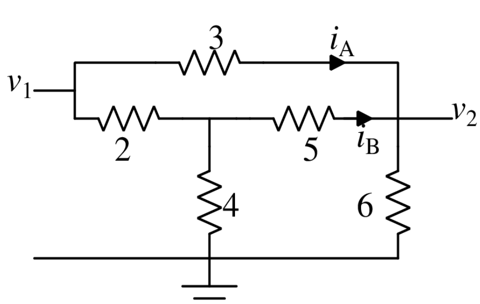
Thus, R in Fig. 39(*a*) can be replaced by two resistances R_A and R_B as shown in Fig. 39(*b*) with the following values

$$R_A = \left(\frac{1}{1-G_E R}\right)R, \qquad R_B = \frac{1}{G_E}. \tag{87}$$

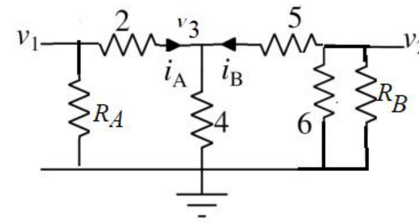
Here also the number of nodes will increase by one.

The MVE was initially called by the name Miller's theorem. Later all others are also called as various versions of Miller's theorems.

EXAMPLE 1
For the circuit shown in Fig. 40(*a*) determine the voltage ratio $A_v = v_2/v_1$ using Miller's theorem.



(*a*)



(*b*)

Figure 40. (*a*) Bridged-T network. (*b*) Equivalent circuit.

Using Miller's transfer voltage equivalent to the resistor 3, the circuit reduces to that shown in Fig. 40(*b*) where

$$R_A = \frac{3}{1-A_v} \quad \text{and} \quad R_B = \frac{3A_v}{A_v - 1}.$$

Solving the circuit by series-parallel reduction technique gives

$$A_v = \frac{v_2}{v_1} = \frac{12A_v}{75A_v - 38} \Rightarrow A_v = \frac{2}{3}. \tag{88}$$

Applying Miller resistance equivalent to the resistor 3, the circuit reduces to that shown in Fig. 40(*b*) where $R_A = 3 + R_E$, $R_B = -R_E$ and $R_E = v_2/i_A$. Analysis of the circuit gives

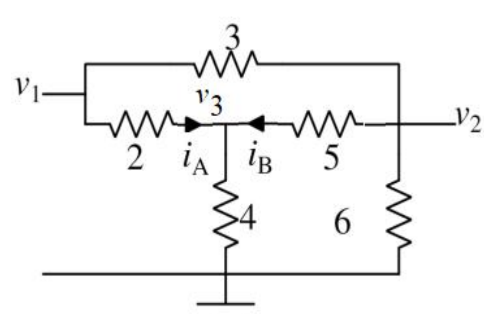
$$R_E = \frac{v_2}{i_A} = \left(\frac{12R_E + 36}{37R_E - 114}\right)R_E \Rightarrow R_E = 6. \tag{89}$$

Or

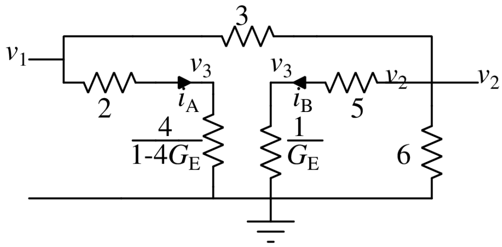
$$\left(\frac{v_2}{(v_1 - v_2)/3}\right) = 6 \Rightarrow \frac{v_2}{v_1} = \frac{2}{3}.$$

From the above example, the following observations are made. Choosing the resistor whose one end has voltage v_2 and the other end has voltage v_1 leads to the quicker solution if the ratio v_2/v_1 is to be determined. The choice of applying MVE to 3-Ω resistor is the ideal choice, as it gives directly the desired voltage ratio as the end result and not the intermediate one.

EXAMPLE 2
In the circuit shown in Fig. 41(*a*) determine the current through the 5-Ω resistor.



(a)



(b)

Figure 41. (*a*) Bridged-T network. (*b*) Equivalent circuit.

Applying Miller's conductance equivalent circuit to the 4-Ω resistor, the resulting circuit is shown in Figure 41(*b*) where $G_E = i_B/v_3$.

Analysis of the circuit gives

$$G_E = \frac{i_B}{v_3} = \left(\frac{-8G_E}{42G_E+6}\right)G_E \Rightarrow G_E = 0 \Rightarrow I_B = 0. \quad (90)$$

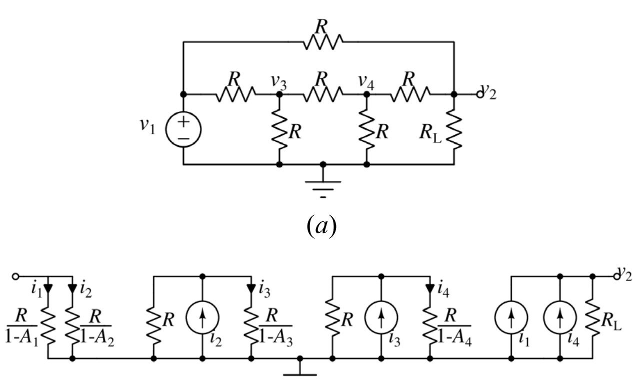
The current through 5-Ω is zero. This is expected as the given circuit is a balanced Wheatstone bridge.

EXAMPLE 3

Determine the voltage ratio v_2/v_1 for the network shown in Figure 42(*a*).

In the previous example we have applied Miller's theorem to one resistor only. However, it can be applied *successively* to other resistors as well. In this problem we shall apply Miller's theorem to more number of resistors *simultaneously*. Applying MVE simultaneously on the three resistors having value R the resulting circuit is shown in Figure 42(*b*).

Let $A_1 = v_2/v_1$, $A_2 = v_3/v_1$, $A_3 = v_4/v_3$ and $A_4 = v_2/v_4$. Obviously



(*a*)

(*b*)

Figure 42. (*a*) Bridged ladder network. (*b*) Equivalent circuit.

$$A_1 = A_2A_3A_4. \quad (91)$$

and

$$i_1 = \frac{v_1}{\frac{R}{1-A_1}},\quad i_2 = \frac{v_1}{\frac{R}{1-A_2}},\quad i_3 = \frac{v_3}{\frac{R}{1-A_3}} \quad (92)$$

and $i_4 = \dfrac{v_4}{\frac{R}{1-A_4}}$

$$v_2 = (i_1+i_4)R_L,\quad v_3 = (i_2-i_3)R,$$
$$v_4 = (i_3-i_4)R. \quad (93)$$

Substituting for i_i, i = 1,2,3,4 and then for v_j, j = 2,3,4 from eqns. (92) and (93) and simplifying we obtain the following equations.

$$(2 + R/R_L)A_1 - A_2A_3 = 1 \quad (94)$$

$$3A_2 - A_2A_3 = 1 \quad (95)$$

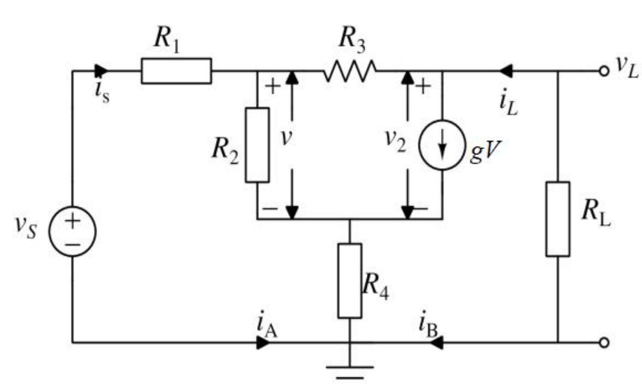
$$A_1 + A_2 - 3A_2A_3 = 0 \quad (96)$$

Solving these equations gives

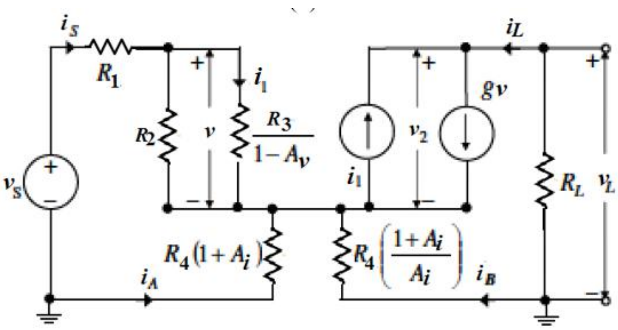
$$A_1 = \frac{v_2}{v_1} = \frac{\begin{vmatrix} 1 & 0 & -1 \\ 1 & 3 & -1 \\ 0 & 1 & -3 \end{vmatrix}}{\begin{vmatrix} 2+R/R_L & 0 & -1 \\ 0 & 3 & -1 \\ 1 & 1 & -3 \end{vmatrix}} = \frac{9R_L}{8R+13R_L}.$$

EXAMPLE 4

Consider the circuit shown in Fig. 43(a). Applying the MVE



(*a*)



(*b*)

Figure 43. (*a*) Active circuit (*b*) Equivalent circuit.

to R_3 and MCE at node C to R_4 simultaneously, the resulting circuit is shown in Fig. 43(b) where $A_i = i_B/i_A$ and $A_v = v_2/v$. From Fig. 43(b),

$$i_1 = \frac{v}{\frac{R_3}{1-A_v}}, \quad v = -i_A\left(R_2 // \frac{R_3}{1-A_v}\right),$$

$$i_L = gv - i_1 \tag{97}$$

$$v_2 = i_B\left[R_L + R_4\left\{\frac{1+A_i}{A_i}\right\}\right]$$

From these equations, we obtain

$$a_1 A_v + b_1 A_i + c_1 A = d_1. \tag{98}$$

$$a_2 A_v + b_2 A_i + c_2 A = d_2 \tag{99}$$

where

$$A = A_v A_i \tag{100}$$

$$a_1 = R_2(R_4 - R_3), b_1 = -(R_2 + R_3)(R_L + R_4),$$
$$c_1 = R_2(R_L + R_4), d_1 = (R_2 + R_3)R_{4,}$$
$$a_2 = R_4, b_2 = -(1 - gR_3)(R_L + R_4),$$
$$c_2 = R_3 + R_L + R_4, d_2 = (1 - gR_3)R_{4,.}$$

Note that $b_2 d_1 - b_1 d_2 = 0$.
From eqns (98) - (100), we obtain

$$A_i = \frac{a_1 b_2 - a_2 b_1}{b_1 c_2 - b_2 c_1} = -\frac{g(R_4 - R_3) + \frac{R_4}{R_2} + 1}{1 + \frac{R_3}{R_2} + \left(g + \frac{1}{R_2}\right)(R_L + R_4)} \tag{101}$$

$$A_v = \frac{d_2 - b_2 A_i}{a_2 + c_2 A_i} = -\frac{(1 - gR_3)(R_4' + R_L)}{(R_L + R_4' + R_3)} \tag{102}$$

where

$$R_4' = R_4\left(\frac{1+A_i}{A_i}\right)$$

Applying potential divider rule at the input and output sides, respectively, gives

$$\frac{v}{v_S} = \frac{R_2 // \frac{R_3}{1-A_v}}{\left(R_2 // \frac{R_3}{1-A_v}\right) + R_1'} = \frac{1}{1 + R_1'\left(\frac{1}{R_2} + \frac{1}{R_3}(1 - Av)\right)}. \tag{103}$$

where $R_1' = R_1 + R_4(1 + A_i)$.

$$\frac{v_L}{v_2} = \frac{R_L}{R_L + R_4'} \tag{104}$$

Voltage gain is easily obtained as

$$\frac{v_L}{v_S} = \frac{v_L}{v_2}\frac{v}{v_S}A_v = \left(\frac{R_L}{R_4' + R_L}\right)\left[\frac{A_v}{1 + R_1'\left(\frac{1}{R_2} + \frac{1}{R_3}(1 - Av)\right)}\right]. \tag{105}$$

III. MATRIX METHOD

The following steps are involved in the matrix method of node analysis [18].

1. The unknown node voltages are identified.
2. The controlled variables are expressed in terms of unknown node voltages.
3. The dependent and independent voltage sources are converted into current sources.
4. The matrix equation [*Y*][*V*] = [*I*] is formulated.

5. The right hand [I] matrix consists of independent and dependent sources. The controlled variables are replaced in terms of unknown node voltages.
6. The right hand matrix is split into two parts: one with independent current sources and the other with unknown node voltage variables as

$$[Y][V] = [I_A] + [X][V]$$

where [X] is a suitable admittance [Y_A] matrix. The second term on the right hand side is brought to the left hand side and subtracted element by element from the [Y] matrix to obtain

$$[Y_B][V] = [I_A]$$

7. This equation is solved for the unknown node voltages using Cramer's rule.

The following steps are involved in the matrix method of loop analysis [18].

1. The unknown loop currents are identified.
2. The controlled variables are expressed in terms of unknown loop currents.
3. The dependent and independent current sources are converted into voltage sources.
4. The matrix equation [Z][I] = [V] is formulated.
5. The right hand [V] matrix consists of independent and dependent sources. The controlled variables are replaced in terms of unknown loop currents.
6. The right hand matrix is split into two parts: one with independent voltage sources and the other with unknown current variables as

$$[Z][I] = [V_A] + [X][I]$$

where [X] is a suitable impedance [Z_A] matrix. The second term on the right hand side is brought to the left hand side and subtracted element by element from the [Z] matrix to obtain

$$[Z_B][I] = [V_A].$$

7. This equation is solved for the unknown loop currents using Cramer's rule.

After sufficient practice some of the steps can be skipped. The method will now be demonstrated with examples.

Example 5: Determine V_2/V_S for the circuit shown in Fig.44

where $R'_S = R_S + r_x$, $Z_\mu = \dfrac{1}{sC_\mu}$ and $Z_\pi = \dfrac{1}{g_\pi + sC_\pi}$.

Let us use the node method. The controlling variable

$$V = V_1 - V_3 \tag{106}$$

The input voltage source V_S is converted into the current source I_S and shown in Fig. 45.

The node equations can be written in a straight forward manner as

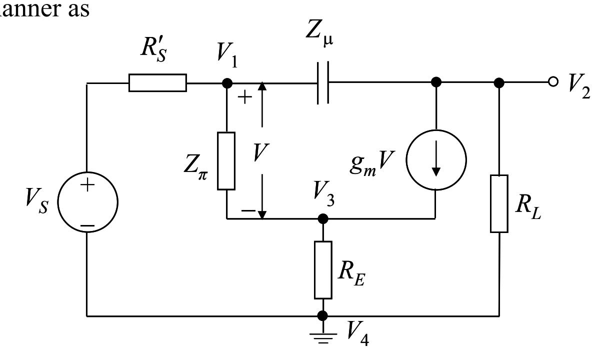


Figure 44. Circuit for example 5.

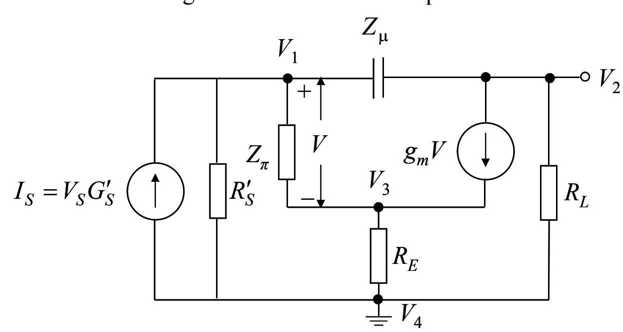


Figure 45. Equivalent circuit after voltage source transformation.

$$\begin{bmatrix} sC_\mu + G'_S + Y_\pi & -sC_\mu & -Y_\pi \\ -sC_\mu & G_L + sC_\mu & 0 \\ -Y_\pi & 0 & G_E + Y_\pi \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_S G'_S \\ -g_m V \\ g_m V \end{bmatrix}$$

or

$$= \begin{bmatrix} V_S G'_S \\ -g_m\left(V_1 - V_3\right) \\ g_m\left(V_1 - V_3\right) \end{bmatrix} = \begin{bmatrix} V_S G'_S \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ -g_m & 0 & g_m \\ g_m & 0 & -g_m \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} sC_\mu + G'_S + Y_\pi & -sC_\mu & -Y_\pi \\ -sC_\mu + g_m & G_L + sC_\mu & -g_m \\ -Y_\pi - g_m & 0 & G_E + Y_\pi + g_m \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_S G'_S \\ 0 \\ 0 \end{bmatrix} \tag{107}$$

where $G(.) = \dfrac{1}{R(.)}$, $Y_\pi = \dfrac{1}{Z_\pi}$.

Note that $Y_{mn} = Y_{nm} \quad (n \neq m)$ due to the presence of g_m. Thus the network is non-reciprocal. From (107) using Cramer's rule we obtain

$$\frac{V_2}{V_S} = G_S' \frac{\begin{vmatrix} sC_\mu + G_S' + Y_\pi & 1 & -Y_\pi \\ -sC_\mu + g_m & 0 & -g_m \\ -Y_\pi - g_m & 0 & G_E + Y_\pi + g_m \end{vmatrix}}{\begin{vmatrix} sC_\mu + G_S' + Y_\pi & -sC_\mu & -Y_\pi \\ -sC_\mu + g_m & G_L + sC_\mu & -g_m \\ -Y_\pi - g_m & 0 & G_E + Y_\pi + g_m \end{vmatrix}} \tag{108}$$

$$= \frac{G_S'\{g_m(Y_\pi + g_m) - (-sC_\mu + g_m)(G_E + Y_\pi + g_m)\}}{\begin{bmatrix} (G_E + Y_\pi + g_m)\{(G_L + sC_\mu)(sC_\mu + G_S' + Y_\pi) + \\ sC_\mu(-sC_\mu + g_m)\} - (Y_\pi + g_m)\{g_m sC_\mu + (G_L + sC_\mu)Y_\pi\} \end{bmatrix}} \tag{109}$$

This relation can also be obtained by using Miller's equivalents in succession as in [14] and simultaneously as explained above [15] but with a cumbersome lengthy algebra. In the present method, the algebra involved is only in solving (108).

Note that if $R_E = 0$, the circuit reduces to that given in Figure 3 of [14]. Here $V_3 = 0$. Hence, from (107) after deleting 3rd row and 3rd column, we obtain

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where

$$y_{11} = sC_\mu + G_S' + Y_\pi,\ y_{12} = -sC_\mu, \quad y_{21} = -sC_\mu + g_m,\ y_{22} = G_L + sC_\mu, \quad I_1 = V_S G_S'. \tag{110}$$

Solving we obtain

$$\frac{V_2}{V_S} = \frac{G_S'}{C_\pi}\left(\frac{s - \dfrac{g_m}{C_\mu}}{s^2 + s\left\{\dfrac{G_L}{C_\mu} + \dfrac{G_S' + g_\pi + g_m + G_L}{C_\pi}\right\} + \dfrac{G_L(G_S' + g_\pi)}{C_\pi C_\mu}} \right). \tag{111}$$

Alternatively, substituting $R_E = 0$ in (109), we can obtain (111) which is the same as given by eqn (12) in [14].

Example 6: Determine V_O/V_S for the circuit shown in Figure 46 by loop analysis. Loop currents are shown.

The controlling variable

$$I_b = I_1 - I_2 \tag{112}$$

Eliminating the controlled current source the circuit reduces to that shown in Fig. 47.

The loop equations can be written in a straight forward manner as

$$\begin{bmatrix} R_S + R_B + h_{ie} + R_E & -(h_{ie} + R_E) \\ -(h_{ie} + R_E) & R_E + h_{ie} + R_C + R \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_S - h_{fe} I_b R_E \\ h_{fe} I_b (R_E + R_C) \end{bmatrix}$$

$$= \begin{bmatrix} V_S - h_{fe} R_E (I_1 - I_2) \\ h_{fe}(R_E + R_C)(I_1 - I_2) \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \end{bmatrix} + \begin{bmatrix} -h_{fe} R_E & h_{fe} R_E \\ h_{fe}(R_E + R_C) & -h_{fe}(R_E + R_C) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

or

$$\begin{bmatrix} R_S + R_B + h_{ie} + R_E + h_{fe} R_E & -(h_{ie} + R_E) - h_{fe} R_E \\ -(h_{ie} + R_E) - h_{fe}(R_E + R_C) & R_E + h_{ie} + R_C + R + h_{fe}(R_E + R_C) \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \end{bmatrix}$$

Solving

$$I_1 = \frac{V_S[R + h_{ie} + (1 + h_{fe})(R_E + R_C)]}{\Delta} \tag{113}$$

$$I_2 = \frac{V_S[h_{ie} + (1 + h_{fe})R_E + h_{fe} R_C]}{\Delta} \tag{114}$$

where

$$\Delta = \begin{vmatrix} R_S + R_B + h_{ie} + R_E(1 + h_{fe}) & -[h_{ie} + R_E(1 + h_{fe})] \\ -[h_{ie} + R_E(1 + h_{fe}) + h_{fe} R_C] & R + h_{ie} + (1 + h_{fe})(R_E + R_C) \end{vmatrix}$$

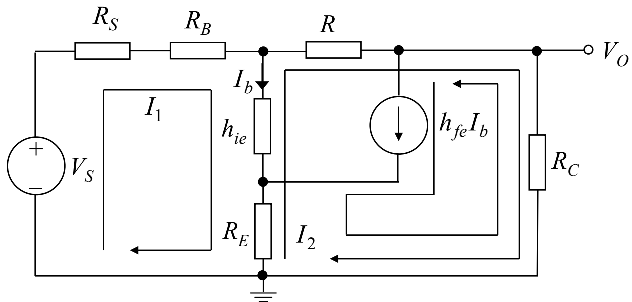


Figure 46. Circuit for example 6.

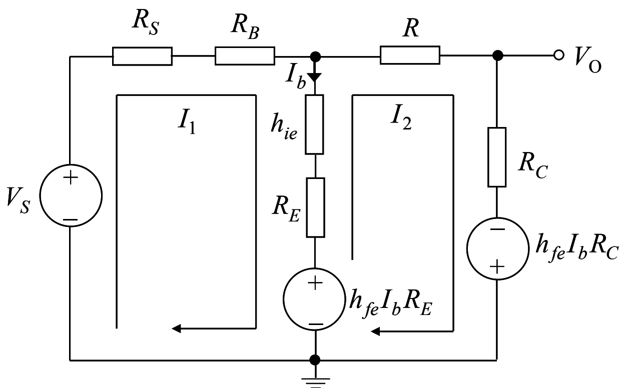


Figure 47. Equivalent circuit after source transformation.

From Figure 47

$$V_O = -h_{fe}R_C I_b + R_C I_2 = -h_{fe}R_C I_1 + R_C\left(1+h_{fe}\right)I_2$$

Substituting for I_1 and I_2 from (113) and (114) and simplifying we obtain

$$\frac{V_O}{V_S} = \frac{-h_{fe}R_C R + R_C[h_{ie} + R_E(1+h_{fe})]}{\left[\begin{array}{l}(R_S+R_B)[R+h_{ie}+(1+h_{fe})(R_C+R_E)]+\\ {[h_{ie}+R_E(1+h_{fe})](R+R_C)}\end{array}\right]} \tag{115}$$

Note that if $R \to \infty$

$$\frac{V_O}{V_S} = \frac{-h_{fe}R_C}{R_S + R_B + h_{ie} + R_E\left(1+h_{fe}\right)} \tag{116}$$

which tallies with the result of example 3 in [14].

Example 7: Determine output admittance Y_{out} and reverse voltage gain A_r for the circuit shown in Fig.48 where z-parameters for the sub-network N_1 are z_{11}, z_{12}, z_{21}, z_{22}.

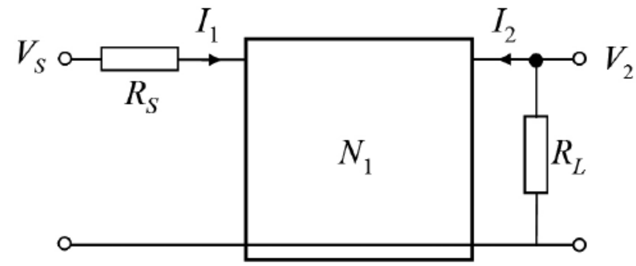


Figure 48. Circuit for example 7.

Here we have

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_S - I_1 R_S \\ V_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} z_{11}' = z_{11} + R_S & z_{12}' = z_{12} \\ z_{21}' = z_{21} & z_{22}' = z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_S \\ V_2 \end{bmatrix}. \tag{117}$$

Output impedance seen by R_L

$$Z_{out} = \left.\frac{V_2}{I_2}\right|_{V_S=0} = \frac{1}{y_{22}'} = \frac{\Delta z'}{z_{11}'} = \frac{\Delta z + z_{22}R_S}{z_{11} + R_S}. \tag{118}$$

This is also obtainable (by a method other than the matrix approach) from the relation given on p 665 of [19]. Now

$$Y_{out} = \frac{1}{Z_{out}} = \frac{z_{11} + R_S}{\Delta z + z_{22}R_S} \tag{119}$$

Thus, including R_L

$$Y_{out} = \frac{1}{R_L} + \frac{z_{11} + R_S}{\Delta z + z_{22}R_S} = \frac{1}{R_L} + \frac{y_{22} + \Delta y R_S}{1 + y_{11}R_S}. \tag{120}$$

Reverse voltage gain

$$A_r = \left.\frac{V_S}{V_2}\right|_{I_1=0} = \frac{z_{12}'}{z_{22}'} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} \tag{121}$$

Now consider the network N_1 as shown in Figure 49 [4]. By the proposed matrix method, or directly from eqn (120) after substituting

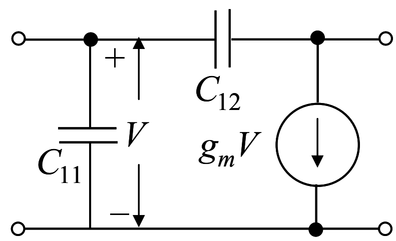


Figure 49. Network N_1.

$$G'_S = 0,\ Y_\pi = sC_{11},\ C_\mu = C_{12},\ G_L = 0$$

we obtain $y_{11} = s(C_{11} + C_{12}),\ y_{12} = -sC_{12},$

$$y_{21} = -sC_{12} + g_m,\ y_{22} = sC_{12}.$$

Substituting for y's, (120) and (121) yield, respectively,

$$Y_{out} = \frac{1}{R_L} + sC_{12}\left[\frac{1+(g_m + sC_{11})R_S}{1+s(C_{11}+C_{12})R_S}\right] \tag{122}$$

and

$$A_r = \frac{C_{12}}{(C_{11}+C_{12})}. \tag{123}$$

Note that eqn (122) is the same as eqn (6) in [4] which is obtained by direct analysis and not by Miller equivalent circuit approach.

Comparison with the method using superposition theorem and matrix method.

There is a similarity between the methods based on superposition theorem [20] and Miller's equivalents. The former method is applicable to the circuits in which sources are dependent on some voltage or current; while the latter is applicable to the circuits in which the elements are dependent on some transfer function. However, in both the methods, one has to determine the controlling variables first and then any other desired voltage or current. We have seen above that the matrix method [18] is more efficient than the method using Miller's equivalents [15].

IV. GENERALIZED MILLER THEOREM

Table 1 gives the Miller equivalent circuits for the four connections of two 2-port networks N_1 and N_2 mentioned earlier. For each connection, N_1 is assumed to have a specific forward transfer function A as given in Table 2.

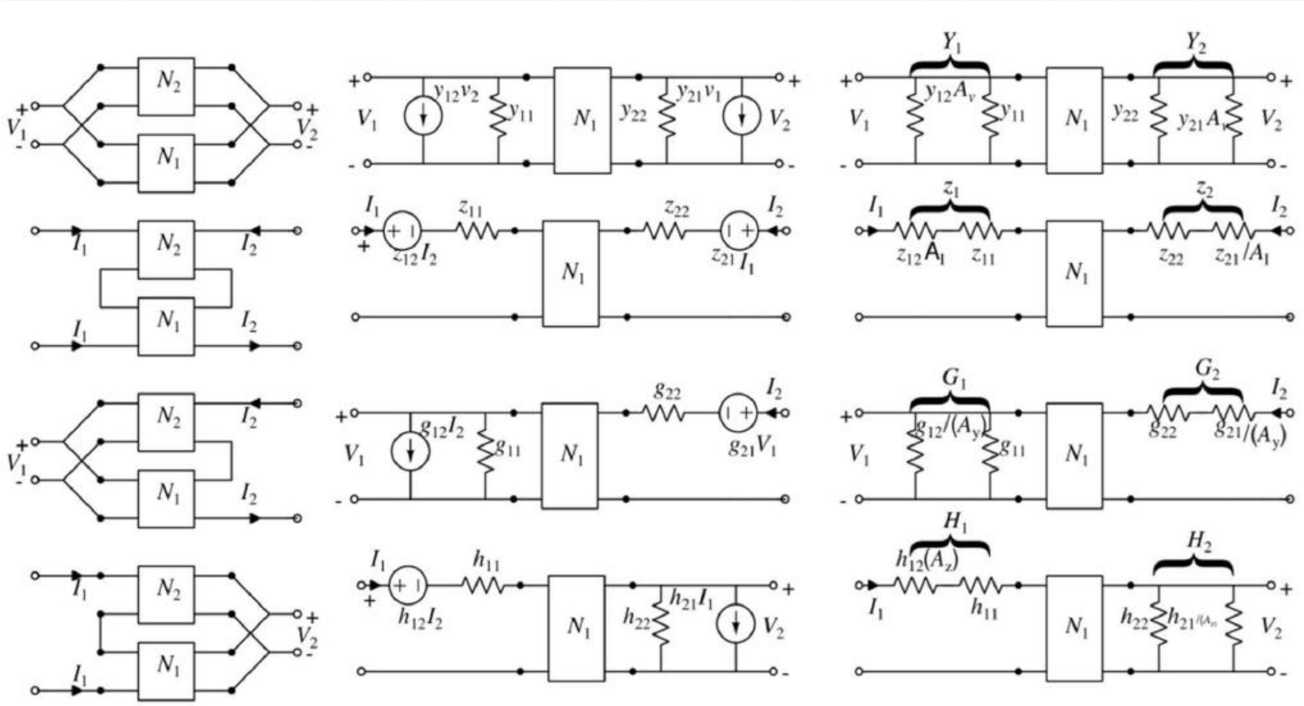
TABLE 2

Connection	A		
PP	Voltage gain	$A_V=V_2/V_1$	(124a)
SS	Current gain	$A_I=I_2/I_1$	(124b)
PS	Transfer admittance	$A_Y=I_2/V_1$	(124c)
SP	Transfer impedance	$A_Z=V_2/I_1$	(124d)

The following general procedure has been adopted in arriving at these Miller circuits.

TABLE 1

Connections	2-Generator equivalent circuit	Miller equivalent circuit



Step 1: N_2 is replaced by its appropriate two-generator equivalent circuit [1] depending upon its interconnection with N_1 as given in Table 3.

Step 2: Each generator is then expressed in terms of the specific forward transfer function of N_1 as shown in Table 4.

TABLE 3

Connection	Equivalent circuit in terms of
PP	*y* parameters
SS	*z* parameters
PS	*g* parameters
SP	*h* parameters

Step 3: These generators are replaced by equivalent emittances using the substitution theorem [17]. The resulting circuits are shown in the last column of Table 1.

TABLE 4

Connection	
PP	$y_{12}V_2 = (y_{12}A_v)V_1$ $y_{21}V_1 = (y_{21}/A_V)V_2$
SS	$z_{12}I_2 = (z_{12}A_1)I_1$ $z_{21}I_1 = (z_{21}/A_1)I_2$
PS	$g_{12}I_2 = g_{12}/(A_Y)V_1$ $g_{21}V_1 = g_{21}/(A_Y)I_1$
SP	$h_{12}V_2 = (h_{12}(A_Z)I_1$ $h_{21}I_1 = (h_{21}/(A_Z)V_2$

Step 4: Two series (parallel) impedances (admittances) are finally replaced by a single equivalent impedances (admittance). We shall call it Miller's emittance. Thus

$$\text{Miller emittance} = \begin{cases} Y_1 = y_{11} + y_{12}A_V & (125a) \\ Y_2 = y_{22} + y_{21} / A_V & (125b) \\ Z_1 = z_{11} + z_{12} / A_I & (126a) \\ Z_2 = z_{22} + z_{21}A_I & (126b) \\ G_1 = g_{11} + g_{12}A_Y & (127a) \\ G_2 = g_{22} + g_{21} / A_V & (127b) \\ H_1 = h_{11} + h_{12}A_Z & (128a) \\ H_2 = h_{22} + h_{21} / A_Z & (128b) \end{cases}$$

From the above theory, we make the following observations.

1. The approach followed here is general and different from that in [3][5][20].
2. Expressions for Miller emittances are simple and easy to remember. Because of the similarity in their expressions, a general Miller theorem can be stated as follows.
   If two 2-port networks N_1 and N_2 are interconnected, Then N_2 can be replaced by two-terminal emittances X_1 and X_2 at ports 1 and 2, respectively, given

$$X_i = x_i + x_i A, \quad I = 1,2; \quad j = 2,1 \tag{128}$$

where forward gain A of N_1 and x are defined Table V.

TABLE 5

Connection	*A*	*x*	X_i
PP	A_V	y	Parallel elements Y_1, Y_2
SS	A_I	*z*	Series elements Z1, Z2
PS	A_Y	g	Parallel element G_1 Series element G_2
SP	A_Z	*h*	Series element H_1 Parallel element H_2

4. PP and SS, PS and SP connections are dual pairs and so also are their equivalent circuits. The Miller equivalents corresponding to the former pair represent the generalized forms of those in [3]. The results of [5] follow after substituting in (127) and (128) the *g* and *h* parameters of the specific networks N_2 used, i.e.,

$$g_{11} = \frac{1}{Z_a + Z_b}, g_{12} = -g_{21} = \frac{Z_a}{Z_a + Z_b},$$
$$g_{22} = \frac{Z_a Z_b}{Z_a + Z_b} \tag{129}$$

and

$$h_{11} = \frac{Z_a Z_b}{Z_a + Z_b}, h_{12} = -h_{21} = -\frac{Z_b}{Z_a + Z_b},$$
$$h_{22} = \frac{1}{Z_a + Z_b}. \tag{130}$$

Thus, Miller equivalents of PS and SP connections in Table 1 represent the generalized forms of those in [5].

6. Effect of N_2 is to modify the driving point immittances of N_1. Thus, the Miller's theorem can be/has been beneficially used in two ways:

i) *Analysis*—Complexity introduced by N_2 network can be reduced by replacing it with equivalent Miller's immittances.

ii) *Synthesis*—Driving point impedance of N_1 can be modified by connecting suitable N_2 network to realize a desired impedance.

In the next section some applications of the Miller's theorem are given to demonstrate its power as an analytical tool in the analysis and synthesis of networks.

V. APPLICATIONS

A. PP Connection

This is the well-known connection which is often simplified by the Miller's theorem. However, it may be pointed out that the analysis becomes simplified only if A_V is either known or can be determined independently. Examples where A_V is known can be seen in [21]. When A_V is not known; the analysis is carried out by assuming approximate value for A_V. For example, in high frequency analysis of common A_V emitter amplifier, a mid-band value of A_V is used as can easily be obtained by inspection [22, p. 468]. Similarly, while analysing the emitter follower [22, p. 474] and Darlington pair circuits, A_V is assumed to be 1. We analyse here a circuit for which A_V is neither known nor assumed. Consider the FET amplifier circuit of Fig. 49(*a*). Its ac equivalent and the simplified circuit after replacing N_2 by Miller impedances are shown in Fig. 50(*b*) and (*c*), respectively, where $A_V = V_2/V_1$. From Fig. 59(*c*), one can easily find that

$$A_V = -\frac{\mu R_2 R_3 A_V}{(r_d + R_1)[(R_2 + R_3)A_V - R_2] + R_2 R_3 A_V}.$$

Solving for A_V

$$A_V = -\frac{\mu R_2 R_3 - R_2(r_d + R_1)}{(r_d + R_1)[(R_2 + R_3)A_V - R_2] + R_2 R_3 A_V}.$$

Input resistance $r_i = \dfrac{R_3}{1 - A_V}$.

The Miller theorem by Miller's theorem has not been explicitly used in synthesis of networks. We demonstrate here some applications in this area. First we take the synthesis of driving point functions. Consider the configuration shown in Fig. 51. After applying the Miller theorem, we find that the input impedance

$$Z_{in} = \frac{Z}{1 - A_V}. \tag{131}$$

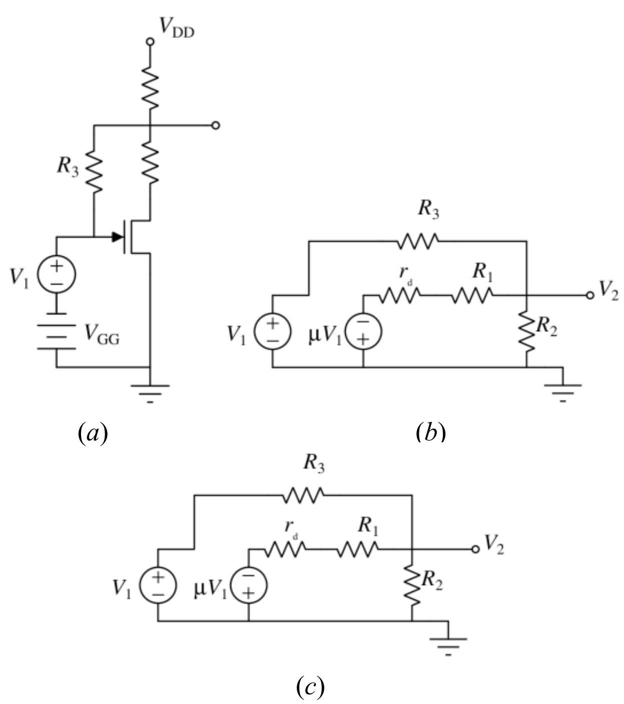


Figure 50. *(a)* FET amplifier, *(b)* ac equivalent circuit, *(c)* simplified circuit.

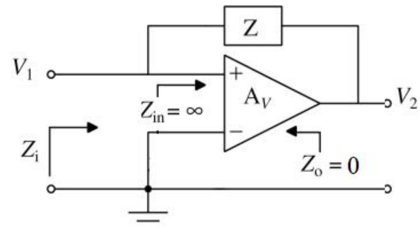


Figure 51. General configuration for realizing driving point functions.

Thus, to synthesize Z_{in} by this configuration, one has to determine suitable Z and A_V. From eqn (131)

$$A_V = 1 - \frac{Z}{Z_{in}}.$$

Split Z_{in} such that $Z_{in} = Z_a Z_{RC}$ where Z_{RC} is RC realizable impedance. Let $Z = Z_{RC}$. Then

$$A_V = 1 - \frac{Z}{Z_a} = 1 - \frac{1}{T}.$$

Now A_V can be realized as follows. Realize $T = Z_a$ as a voltage transfer function by suitable active-RC network. Then carry out τ_{OI} and τ_{IE} operations [23] in sequence or a τ_{OEI} operation in one shot on T to realize A_V. In the former case, if $T = Z_a$ realization happens to be a chain of n cascaded networks having voltage transfer function $T_1, T_2, \ldots, T_n$, a number of A_V realizations are possible. This is due to the fact that the reciprocal of T can be expressed as a product of the reciprocals of the voltage
impedance. Let $Z = Z_{RC}$. Then

$$A_V = 1 - \frac{Z}{Z_a} = 1 - \frac{1}{T}.$$

transfer functions of a chain of subgroups of the entire circuit. For instance,

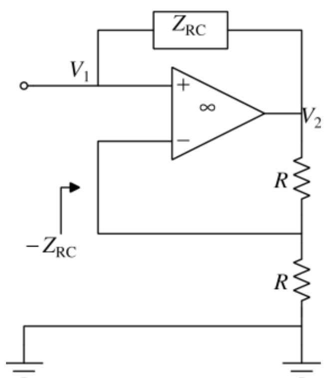
$$\frac{1}{T} = \left(\frac{1}{T_1 T_2 T_3 \ldots. T_n}\right) = \left(\frac{1}{T_1 T_2}\right)\left(\frac{1}{T_3}\frac{1}{T_4}\cdots\frac{1}{T_n}\right) = \ldots.$$

Following the above procedure, realization of an NIC, a *C*-multiplier, an ideal inductor, and an FDNR (frequency-dependent negative resistor) are given in Fig. 51(*a*). Realization steps are summarized in Table 6.

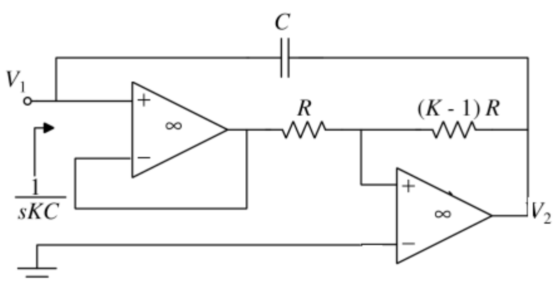
Table 6

	NIC	C-Multiplier	Inductor	FDNR
Z_{in}	$-Z_{RC}$	$1/KsC$, $K > 1$	s	s^2
Z	Z_{RC}	$1/sC$	1	1
$T=Z_o$	1	$1/K$	s	s^2
A_V	2	$-(K-1)$	$1-1/s$	$1-1/s^2$

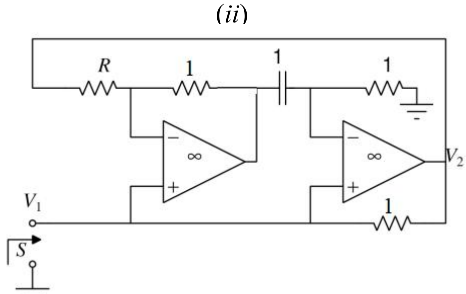
In the inductor realization, we realized $T = s$ by an inverting differentiator preceded by an inverter. In the case of FDNR, $T = s^2$ has been realized by a cascade connection of two non-inverting differentiators. In both cases, A_V has been obtained by a T_{OEI} operation [23]. Other alternative realizations involving τ_{OI} and τ_{IE} operations are shown in Fig. 52(b).



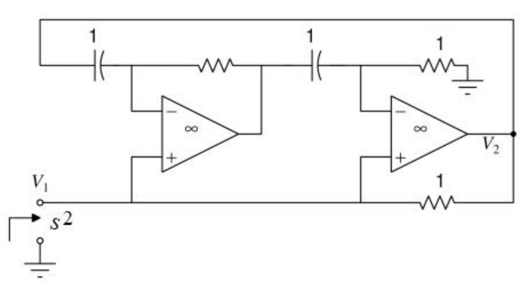
(i)



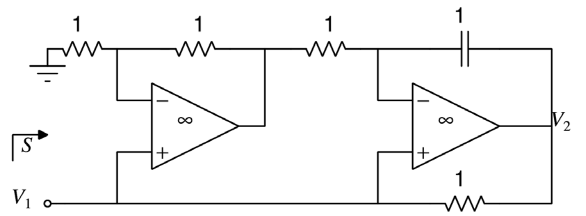
(ii)



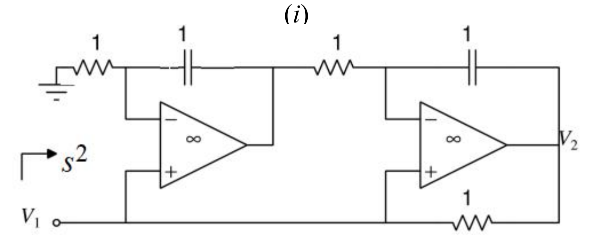
(iii)



(iv)
(a)



(i)



(ii)
(b)

Figure 52. (a) Realization of (i) NIC, (ii) C multiplier, (iii) ideal inductor, (iv) FDNR, (b) Additional realizations of (i) inductor, (ii) FDNR.

In the case of an inductor, two more realizations can be obtained by interchanging the order of inverter and differentiator in T realization in Fig. 52.

Now we consider the synthesis of transfer functions. Since the open-circuit voltage transfer function of the RC network N_A in Fig. 53 is $T = -y_{21A}/y_{22A}$, the poles of T are restricted to the negative real axis only. However, if y_{22} is modified by adding another term, it may be possible to change the pole positions. This extra admittance term is provided by the Miller admittance reflected by the network connected to N_A as shown in Fig. 53. Poles may be shifted to the desired position by choosing the N_B network and A_V. Kuh [25, p. 311] and the other structure [25, p. 311] are the special cases of the configuration of Fig. 53.

Now consider the circuit shown in Fig. 54. The short-circuit current transfer of the RC network N_A is $-y_{21A}/y_{11A}$. Thus, the poles are restricted to the negative real axis. However, the Miller Admittance Y_1, reflected by the remaining portion of the circuit in Fig. 54, changes y_{11} and thus modifies the pole locations of the overall current transfer function. For simplicity of the synthesis, N_B in both Fig. 53 and 54 may be taken as a simple series admittance Y. The exact synthesis procedure can be seen in [24, p. 322]. Similarly, many other configurations, such as those given in [24, ch. 8], can also be

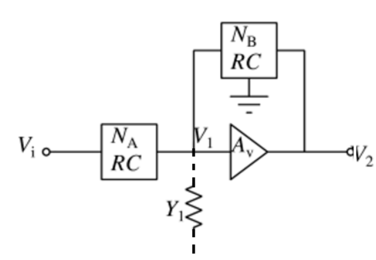


Figure 53. Active RC realization of voltage transfer function.

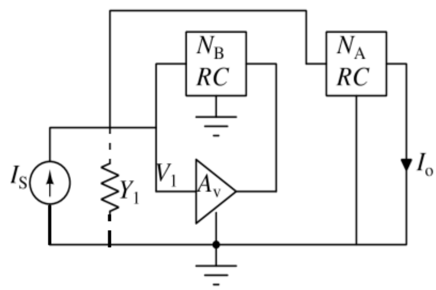
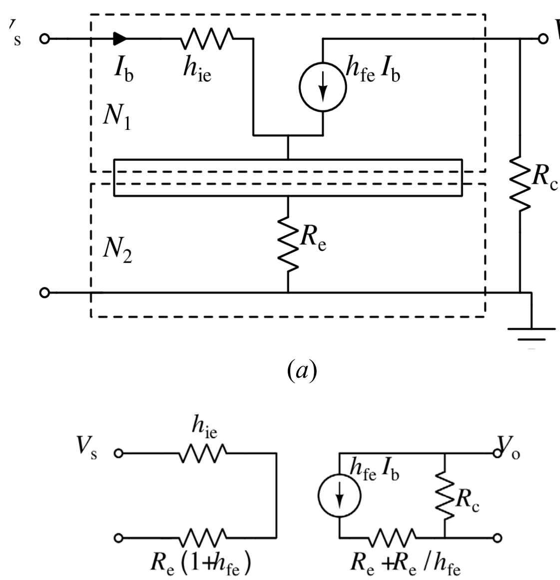


Figure 54. Active RC realization of current transfer function.

analysed through the Miller's theorem to see how poles and / or zeroes relocated.



(b)

Figure 55. (a) Equivalent circuit of common emitter amplifier, (b) Reduced circuit.

B. SS Connection

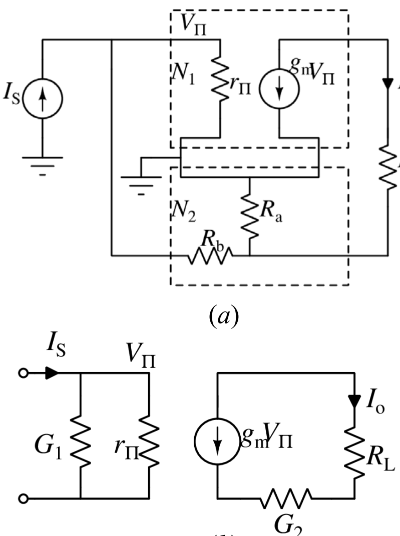
Consider the ac equivalent circuit of a common emitter amplifier shown in Fig. 55(a); the reduced circuit after application of the Miller's theorem is shown in Fig. 55(b).

Thus, the Miller theorem explains the reflection of $(1 + h_{fe})$ times larger input resistance of what is connected in the emitter leg. In a similar manner, one can easily verify that in common base amplifier, any resistance R_B connected in the base leg is reflected as $R_B/(1 + h_{fe})$ in the input circuit.

C. PS Connection

We have taken the circuit shown in Fig. 56(a) from [5]. After applying the Miller's theorem, it reduces to that shown in Fig. 56(b). Simple analysis leads to the result the same as obtained in [14].

$$\frac{I_o}{I_S} = -\frac{1 + R_b / R_a}{1 + \frac{1 + R_b / R_a}{g_m r_\pi} + \frac{1}{R_a g_m}}. \tag{136}$$



(b)

Figure 56. (a) PS connection, (b) Reduced circuit.

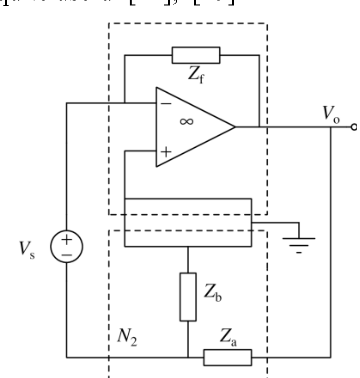
D. SP Connection

An example of this type of connection is shown in Fig. 57(a) and (b) after applying the Miller's theorem. By inspection,

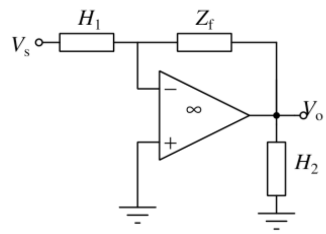
$$\frac{V_O}{V_S} = -\frac{Z_f}{H_1} = -\frac{Z_f(Z_a + Z_b)}{Z_b(Z_a + Z_f)}. \tag{137}$$

If $Z_a \to \infty$, it reduces to more familiar result $V_0/V_S = -Z_f/Z_b$ of an inverting *OA* amplifier.

From the above applications, we see that the Miller's theorem drastically simplifies the analysis if a specific transfer function A of N_1 is known However, in the analysis of feedback amplifiers, the specific transfer function A is usually not known. In such cases, the two generator equivalent circuits of Table 1 are quite useful [21], [25]



(a)



(b)

Figure 57. (a) SP connection, (b) Reduced circuit.

The left-hand generator represents the feedback signal and the right handed generator represents the feedforward signal. Thus, the feedback factor β is y_{12}, z_{12}, g_{12} or h_{12} depending upon the connection.

VI. CONCLUSION

Four general Miller equivalent circuits, one for each of the four possible connection of two two-port networks, have been derived. The earlier known versions [3], [14] are the special cases of these circuits. Moreover, the method followed here for deriving them is different from ones. A generalized Miller theorem has been stated. Typical applications are included to demonstrate the analytical power of the Miller theorem in the analysis and synthesis of networks.

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